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# Sustainable Markov Chain Models for Telemedicine

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**Abstract.** Communication systems are made of reliable components, both hardware and software, and therefore they may have only a few failures in exploitation. For an application field such as telemedicine this desiderate is a must. We model these systems dedicated to telemedicine with randomized pulse modulation data traffic using a finite Markov chains. Based on previous assumption telemedicine communication systems failures are considered, and therefore modelled using the rare event with a finite-state formalism. Our work is focused on improving the telemedicine systems` reliability using randomized pulse width modulation.

**Keywords:** Telemedicine, Markov chains, pulse width modulation, rare events, discrete event systems

## 1 Introduction

To estimate patterns of telemedicine communication systems we deal with discrete event models of their reliability based on observed success/failure data as we noticed in practice. As noticed in literature, failures are rare events, in telemedicine communications [1-3], and therefore we assume that their occurrence probabilities are smaller by at least several orders of magnitude than probabilities of ordinary failure events. Therefore, the mean time to failure (MTTF) one of the most used reliability parameter, is a considerable number. User friendly representation and computations characterise the Markov chain for modelling and analysing rare events. The Markov chain provides also a direct measure of rarity using steady-states probabilities. Here we deal with a discrete-parameter, finite-state Markov chain [4, 5] used for representing both data communications failure (as transitions to a rare fail-state) and randomized pulse width modulation schemes of transmitter. We choose this approach as it offers few advantages, such as: the randomized modulation of switching reduces filtering equipment's and allows an explicit time domain control, and is reported in literature, randomized modulation is effective for dealing with narrow band constraints [6]. The remainder of this paper is as follows: Section 2 describes the basic principle of the proposed data transmitter and receiver, Section 3 deals with our approach for Markov chains modelling formalisms of randomized modulation availability. Section 4 analysis the sustainability of the proposed Markov model, while Section 5 exemplifies a randomized modulation modelled with periodic Markov chains, and proposes a new model of periodic Markov chain. Section 6, based on an illustrative example, analysis the reliability of this approach here entitled reliability Markov chain model (RMCM). Conclusion synthesizes the paper, the proposed model and display few development slopes.

## 2 Proposed Data Structure

Basically, a data transmitter involves generating a switching function f(t), which for example is equal to “1” when the switch is conducting and the “0” otherwise. This is schematically represented in Fig. 1. We assume that for value “1” there is some medical data to be transmitted; respectively there is one package (which includes several distinct diagnoses, images, medication etc., each one embedded in a sine wave) of data to be transmitted, as illustrated in Fig. 2. For verifying the correctness of the received data, the decoder counts the number of sine waves periods which fits the length of the pause. The resulted number, which must be a par multiple of the sinusoids (e.g, a par multiple is needed to verify if one sinusoid is missing due to perturbations), indicates the specific number and the sort of data inside the transmitted package, such as “n” medical analyses for patient John Doe. As shown in Fig. 2 the number of the codified pulses g(t) determines the switching function f(t) of the data transmitter [4]. Therefore, the data security is double checked and protected against electromagnetic interferences, as well against data loss or attenuation throughout passing reactive circuits.

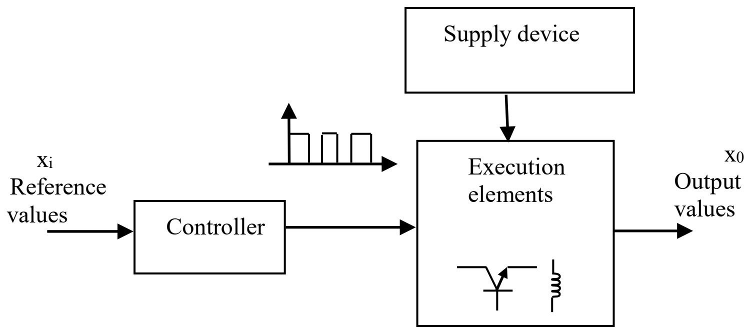


Fig.1 Schematic representation of the proposed switching transmitter

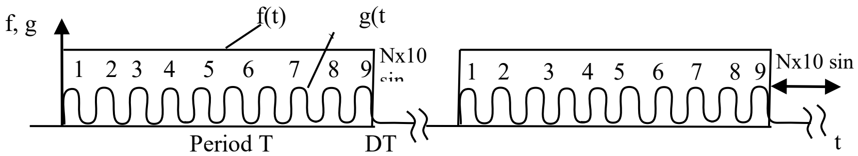


Fig.2. Switching functions of packages of data traffic in telemedicine

It is stated in literature [4-8] that the average value or duty ratio D for f(t) usually determines the nominal output of a transmitter, as wave forms that are periodic have spectral components only at integer multiples of the fundamental frequency. We noticed that an effort was directed toward the optimization of deterministic PWM waveforms, and alternative in the form of randomized modulation for d.c./a.c. and a.c./a.c. conversion is based on schemes in which successive randomizations of the periodic segments of the switching pulse train are statistically independent and governed by probabilistic rules. These schemes are denoted in literature as stationary [3]. Next chapter describes an approach for the synthesis of this class of stationary randomized modulation schemes that enables explicit control of the time-domain performance of the data traffic applied in telemedicine.

## 3 Estimating Switching Availability with Markov Chains

Telemedicine data transmission process deals with the switching code system availability to design a randomized switching procedure that minimizes given criteria for spectral characteristic of f(t). For estimating the data transmission system availability, we introduce a failure state due to imperfect data coverage (denoted here by "c") and repair (denoted here by "r") [4]. To explain the impact of data imperfect coverage we consider the coding system in Fig. 3 which includes two identical switching codes device (SWCD).

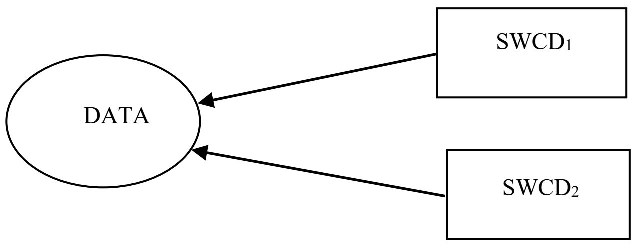


Fig.3 Example of perfect coverage data with two identical codes

For exemplifying the coverage of a transmission data system, we notice that if the coverage of the system is sufficient, e.g. c = 1, then data is transmitted correctly as long as one of the $SWCD_i$ is operational, where i = 1, 2. If the coverage is insufficient, then data is incorrect with probability 1 – c (i.e. we may say that data fail

with probability 1 – c). The Markov chain for modelling data using $SWCD_i$, where i = 1, …, k, with k being a nonnegative integer is shown in Fig. 4, where parameters λ, μ, c, r denotes respectively the failure code rate, repair code rate, coverage data rate, and the successful data repair rate of SWCD.
The first part of the horizontal transition rate with term 1 – c represents the data (e.g. code) failure due to imperfect coverage of an alternative SWCD. The second part, with the term 1 – r represents imprecise repair of SWCD.

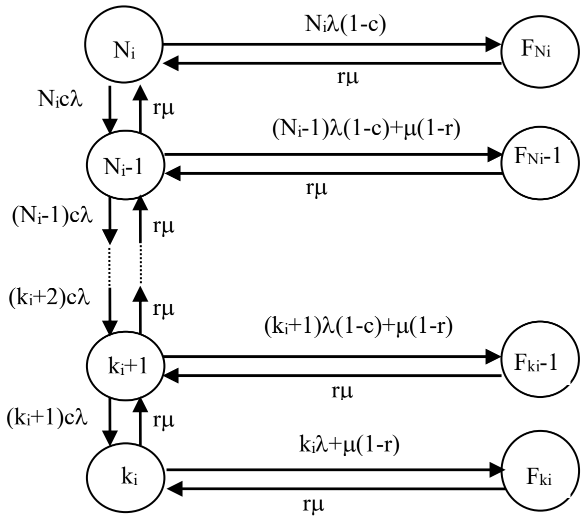


Fig.4. Markov model for $SWCD_i$

The down/up arcs model the failure/repair of $SWCD_i$. As we deal with rare events, we say that only one code fails at a time in a $SWCD_i$. In Fig. 4 on state $N_i$ the $SWCD_i$ is functioning with all $N_i$ codes operational. The trajectory of our system, respectively the states of $SWCD_i$ changes from one of the $k_i$ working states to one of the $F_{ki}$ codes failure states due to imperfect coverage (1 - c), or due to imperfect repair (1 - r). We notice that perfect fault/repair coverage of the system reduces the Markov chain to one-dimension model. The solution of the Markov chain model given in Fig. 4, and implicitly the solution for system functionality is given by the probability that at least $k_i$ codes are correct at time t. The availability of $SWCD_i$ is determined using the following relation [20]:

$$A_i(t) = \sum_{k_i}^{N_i} P_{k_i}(t), \quad \text{for } i = 1, 2, \ldots, n \tag{1}$$

where, $A_i(t)$ = the availability of $SWCD_i$ at time t; $N_i$ = the total number of codes in $SWCD_i$; $k_i$ = required minimum number of operational codes in $SWCD_i$.

After a Markov chain for $SWCD_i$ is built and desired probabilities $A_i(t)$, i = 1, 2, … ,n are determined, the system`s availability (e.g. the switching code system) is given by the following relation:

$$A(t) = \max A_i(t) \qquad (2)$$

To optimize the data transmission is mandatory to minimize the discrete spectral components (denoted as narrow-band optimization), and to ensure minimization of signal power in a given frequency range (denoted as wide-band optimization) [21]. We assume that our Markov model goes through a sequence of n classes of states $C_i$, occupying a state in each class for an average time $\sigma_i$, i = 1, ..., n.
The time-average autocorrelation of the random process f(t) is defined as [6]:

$$R_f(\tau) = \lim_{n \to \infty} \frac{1}{2w} \int_{-w}^{w} E\,[f(t) \cdot f(\tau + t)] \cdot dt \qquad (3)$$

where the expectation E[.] refers to the whole ensemble [.].

The Markov chain of class $C_k$, with the time-averaged autocorrelation (3) is scaled by $\tau_k / \sum_{i=1}^{n} \tau_i$ , where $\tau_i$ the expected time is spent in the class $C_i$ before a transition into the class $C_{i+1}$. We define the (n x n) state-transition matrix P with (k, i)th entry given by the probability that at the next transition the chain goes to state i, given that it is currently in state k. P is a stochastic matrix, so it has a single eigenvalue $\lambda_i = 1$, with corresponding eigenvector $1_n = [1\ 1\ ...\ 1]$, and all other eigenvalues less than 1. In [1] it is proven that after a possible remembering of the states, the matrix P has a block-cyclic form:

$$P = \begin{bmatrix} 0 & P_{12} & 0 & \dots & 0 \\ 0 & 0 & P_{23} & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & P_{n-1,n} \\ P_{n1} & 0 & 0 & \dots & 0 \end{bmatrix} \qquad (4)$$

Let $P_P$ denote the product of submatrices of P: $P_P = P_{n1} \ldots P_{23}. P_{12}$, and let $v_i$ denote the vector of steady-state probabilities, conditional on the system being in class $C_i$. We have:

$$V_i{}^* = v_i \cdot P_P \qquad (5)$$

The time spent in class $C_i$ (maintenance time) is:

$$\tau_i = \sum_k V_k^* \cdot \tau_k \qquad (6)$$

Let $T_o = \sum_{i=1}^{n} \tau_i$ and let $\Theta_i$ = diag ($V_i$*). If the first data string belongs to the class i, then the pulse $\tau_i + \tau$ belongs to the class (i + m)/n, where m represents the number of transitions between data strings between moments $\tau_i$ and $\tau_i + \tau$. We have a case with average duration of some classes' null, which means that these classes are avoided [20-22]. This construction deals with a simplified Markov chain model (we present this assumption, which we believe to be novel, in the next section). When we add the contribution of all classes to the average power spectrum (scaled by the relative average duration of each class) the result can be written as follows [4]:

$$S(f) = \frac{1}{T_0}\left[\sum_{i=1}^{n} \frac{\tau_i}{T_0} \cdot U_i^T(f) \cdot \Theta_i U_i(f) + 2R_e\left(1_n^T \cdot S_C \cdot 1_n\right)\right] + \frac{1}{T_0^2} R_e\left(1_n^T \cdot S_d \cdot 1_n\right) \sum_{i=-\infty}^{\infty} \delta\left(f - \frac{i}{T^*}\right) \quad (7)$$

Where T* is the greatest common divisor of all waveform duration, $1_n$ is an n x 1 vector of 1 and $U_i$ is the vector of Fourier transforms of waveforms assigned to states in class $C_i$. The matrix $S_{c,i}$ has a Toeplitz structure, with (k, i)$^{th}$ entry:

$$S_{ck,i}(f) = \frac{\tau_i}{T_0} \cdot U_k^T(f) \cdot (I - \Lambda_k(f))^{-1} \cdot \Lambda_{k,i}(f) \cdot U_i(f) \quad (8)$$

where $\Lambda_k$ is a product of n matrices: $\Lambda_k = Q_{k-1,k}, \ldots, Q_{k,k+1}$, and $\Lambda_{k,j} = Q_{k,k+1}, \ldots, Q_{i-1,i}$ . Where Q is a matrix n x n whose (k, i) entry is $Q_{k,i}(\sigma) = P_{k,i}\delta(\sigma - \tau_k)$. Also, the (k, i)th entry of $S_d$ is given by relation:

$$S_{dk,i}(f) = \frac{\tau_i}{T_0} \cdot U_k^T(f) \cdot V_k \cdot (V_i)^T \cdot U_i(f) \quad (9)$$

Starting from the notion of truncated Markov chains with absorbing states [6], we propose a simplified model of Markov chains for random modulation. The proposed Markov chain $X^{(m)}$ has the states {m, m+1, …} aggregated into an absorbing class of states, with the transition matrix $_{(m)}T$:

$$_{(m)}T = \begin{bmatrix} _{(m)}P & p_m \\ 0 & 1 \end{bmatrix} \quad (10)$$

Where $p_m = \left[\sum_{j\geq m} p_{1j}, \sum_{j\geq m} p_{2j}, \ldots, \sum_{j\geq m} p_{m-1,j}\right]$.

Since ${}_{(m)}P$ is irreducible for all m, it follows that $X^{(m)}$ constitutes an irreducible Markov chain for all m, where m is an absorbing class of states [m]. The n-step transition matrix ${}_{(m)}T^n$ is:

$$ {}_{(m)}T^n = \begin{bmatrix} {}_{(m)}P^n & \left(I_m + {}_{(m)}P + {}_{(m)}P^2 + \ldots + {}_{(m)}P^{m-1}\right)p_m \\ 0 & 1 \end{bmatrix} \tag{11} $$

Where ${}_{(m)}P^n = {}_{(m)}p_{ij}{}^n = p_{ij}(n)$.

The transition probability between realizable classes of states is 1. The transition matrix probability also indicates the priorities between the states of the system.

## 4 Framing the Sustainability of the Proposed Markov Model

One of the most important parameters that control the performance of a modern system is its reliability. Reliability is measured by the fault exposure ratio (FER) [9-11]. It represents the average detectability of the faults in the system. Other parameters that control FER are the size of the system and the execution speed of the control unit, which are both easily evaluated. We notice that, usually in the literature, the cell loss problem happens in communication networks, and we extend this issue to telemedicine systems, as they use electric energy, and are partially controlled by human operators. Therefore often, the failure probability of an individual component is very small for a well-managed system such as these mentioned above. For instance, the fault probability of a system component for telemedicine is usually at the magnitude of $10^{-6}$ per hour, or less than 1% per year [12]. Based on this idea, our study focuses on these low probability issues mainly, and we may say that although not stated clearly, much of the literature implicitly applies the rule of rare event approximation. Although the applications of our approach are not limited to the discrete time case, we focus our discussion on discrete event models in this paper [13-15]. We see the Markov model given in Fig.4 as a discrete time server that can service *c* cells during one-time unit. This server serves a queue with a capacity for *k* cells which is fed by an independent traffic source. The arrival process associated with a source has two states: active (ON) and idle (OFF), represented by 1 and 0 respectively. In the active state, an arrival can occur with probability $\alpha$ (in the experiments, $\alpha$ is assumed to be 1). No arrivals occur while the source is in the idle state. Each of these ON-OFF sources behave as follows: while an arrival process is in state 0, there is a probability $1-p_{00}$ that will change to state 1 at the next time slot and a probability $p_{00}$ that will remain in state 0. While an arrival process is at state 1, there is a probability $1-p_{11}$ that it will transit to the idle state at the next time slot and a probability $p_{11}$ that it will remain in state 1. When the server is busy, a maximum of *c* cells will depart the system at each time slot. The system can be modelled as a discrete time Markov chain with state $(x_i, y_i)$, where $x_i$ is the number of cells in the queue and $y_i$ is the number of arrival sources in the active state at the $i^{th}$ time slot. We want to determine the steady state behaviour $(x,y) \overset{\Delta}{=} \lim_{i\to\infty}(x_i, y_i)$. Let $S$ denote the state space. Let T = $[t_{n,m;k,l}]$ be the transition matrix for this Markov chain, where $t_{n,m;k,l}$ = Prob[$x_{i+1}$=k, $y_{i+1}$=l | $x_i$=n, $y_i$=m]. Note that the dimension of this Markov chain is (k+c)·(n+1).

In the literature the resources cell loss distribution $P_L$ can then be calculated as in [16]:

$$P_L = \frac{\sum_{(n,m)\in S} \max[(n+m-k),0]\cdot P[x=n, y=m]}{\sum_{(n,m)\in S} m\cdot P[x=n, y=m]} \tag{12}$$

We notice that when the size of the model (either the buffers size, or the number of sources $n$, or the number of processing units $P_i$) becomes large, the computational cost is prohibitively high due to the size of the state space. We consider that such an approach cannot represent the resources cell loss distribution $P_L$ among the $k$ tasks of the considered system.

In order to achieve this goal, we propose a new approach for determining $P_L$. We consider the next function:

$$P_{Lu}(t) = \sum_u \sum_n (-1)^u \cdot (n_u^\alpha + \frac{1}{n-1}) \cdot t^{\frac{n}{c\cdot v}} \tag{13}$$

Where: u = the task accomplished by the system;

t = time required for accomplishing the process;

$c_v$= the coverage factor (e.g. the coverage probability) for the system supplied by $n$ sources through the input buffers;

$n^\alpha{}_u$ = is a function of probability α that the system receives in the $u^{th}$ task the resource number $n$.

We mention that the graphical representation of the function $P_{Lu}(t)$ given in the relation (13) allows to show the loss probability both for odd and even tasks according to the factor $(-1)^u$, respectively by positive and negative inflexions of the network.

The steps required to calculate the rational interpolants for $P_L$, or $P_{Lui}(t)$ are the following ones:

1. Asymptotic analysis: we suppose that log $P_L \approx \Theta$*k ($k \to \infty$), respectively (n → ∞).

We calculate exponential decay rate $\Theta$ using the algorithm proposed in [12].

2. Determine the forms of transformation and the form of approximant sequence. We develop approximants for the function h(k) = log $P_L$(k), respectively h(k) = log $P_{Lui}$(n) and will use an $R_{(n+1),n}$ sequence of rational interpolants since h(k) and h(n) are asymptotically linear [14-15].

3. Evaluate $P_L(k)$, respectively $P_{Lui}(n)$ for small values of $k$, respectively $n$ (thus the corresponding values of h(k) and h(n) are known) by solving the Markov chain or using other available analytic methods (including Henstock integrals).

4. Calculate rational interpolants, $R_{(n+1),n}$ with increasing orders ($n$ = 1, 2, …) and stop when the successive interpolants are sufficiently close to the range of k or $u_i$ of interest.

## 5 Illustrative Example

In this section we deal with an illustrative model applicable for the data transmitter`s code depicted in Fig. 2 in order to generate a switching function where blocks of pulses have deterministic duty ratios: [0.75, 0.5, 0.25]. The periodic Markov chain shown in Fig. 3, with six states divided into three classes, is an example of a solution to this issue [6, 7]. A short cycle (duration 3/4) and a long cycle (duration 5/4) are available in each of the four classes. According to the theoretical approach in the previous paragraph, we build a simplified Markov chain in Fig. 4. The Markov chains in Fig. 5, respectively in Fig. 6 have the same transition probabilities between states $S_i$, i=1,…., 6 of classes $C_j$, j=1,…, 3. One can observe that the Markov chain in Fig. 6 is more intuitive and tidy than the one in Fig. 5. The transition probabilities equal to 1 in Fig. 6 are conditioned by the existence of transition between the states of different classes.

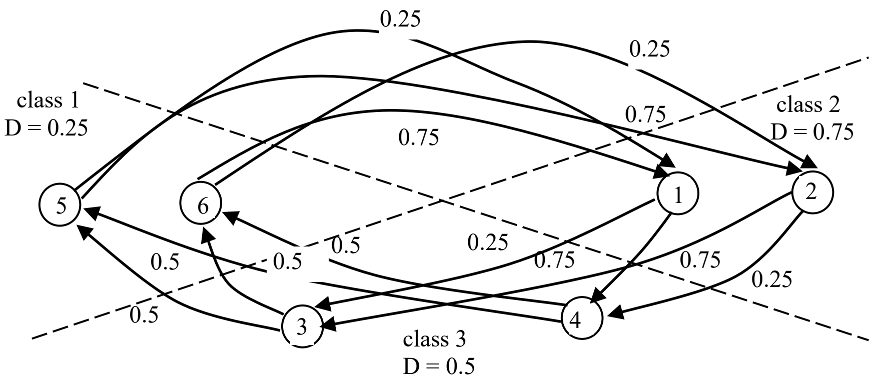


Fig.5 Classic Markov chain for modeling the switching example

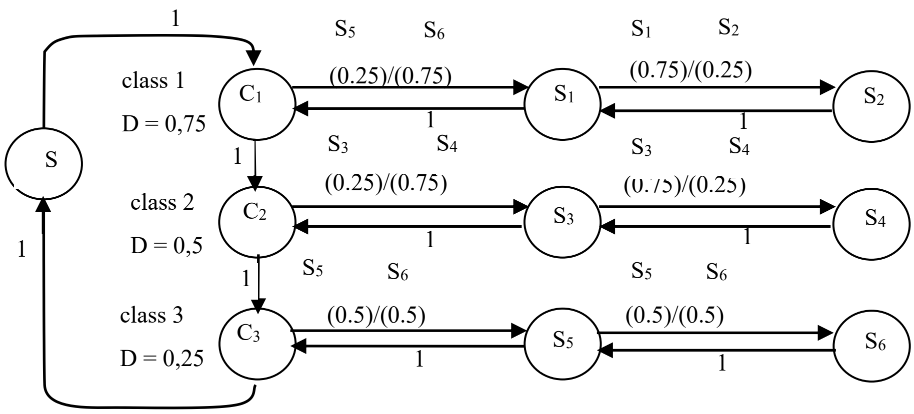


Fig.6. Proposed Markov chain for modeling the switching example

We analyze this Markov chain with the help of equation (4) and we compare the theoretical predictions with the estimates obtained in Monte Carlo simulations. The agreement between the two is quite satisfactory: the theoretical prediction for the impulse strength at f = 4 is 0.0035, and the estimated value is 0.0037. The Markov chain in Fig. 4 allows dealing with many more classes as graphical representation is simplified and can significantly improve the tractability of the optimization of the multi-state Markov chains.

## 6 Reliability Estimation of our Model

The Markov chain for estimating the reliability of our model has at least three states: starting-state S, working state W, and fail state F. State sequences are realizations of reliability Markov chain model (RMCM) [7-8], [22]. A realization from S to first occurrence of W represents a single successful execution cycle of data transmission. A transition from any state to F represents a failure of data transmission. The probabilities on arcs in RMCM are the values estimated for the usage profile and component reliabilities expected in practice [9]. We notice that if state i $\epsilon$ RMCM, and i $\neq$ F, has been visited $n_i$ times and exited without failure, then the probability of failure at state i is no greater than $1/(n_i + 1)$. Let random variable $n_F$ be the number of visits to F in a randomly generated realization of n transitions starting in a state S. Let $\lambda$=E($n_F$) be the mean value of the probability low of $n_F$. Let P = [$p_{ij}$] denote the RMCM's transition probability matrix. The RMCM have all states reachable from S by traces with nonzero probability and arcs from both F and W to S with $P_{FS} = P_{WS} = 1$, so that a successful or unsuccessful path terminated in W, respectively in F state causes an immediate restart in initial state S. RMCM's steady-state probability distribution $\Pi = [\pi_S, \dots, \pi_F]$ is the unique solution of $\Pi = \Pi P$ where $\Sigma\, \Pi_i = 1$ and $\pi_i > 0$ is the limiting relative frequency of occurrence of state i as a count transition, e.g., recurrence time [4], [23, 24]. Adopting a Poisson law with parameter $\lambda$, developments in small number laws stress that $P_0(\lambda)$ is an approximation and compute an upper bound for measuring the distance between $P_0(\lambda)$ and the probability law L($n_F$). The total variation distance $d_{TV}$ [L($n_F$), $P_0(\lambda)$] is defined as show in relation (14) for events A in the sample space [17], where $P_0(\lambda)$ is the Poisson distribution with parameter $\lambda$:

$$d_{TV}[L(n_F), P_0(\lambda)] = \sup_A |L(n_F)(A) - P_0(\lambda)(A)| \tag{14}$$

Since $\Pi_F$ equals the limiting relative frequency of state F, for large n we have:

$$\Pi_F \approx \frac{E(n_F)}{n} = \frac{\lambda}{n} \tag{15}$$

We may approximate for large n, and rare state F:

$$L(n_F) \approx \frac{(n\Pi_F)^k}{k!} \cdot e^{-n\Pi_F} \tag{16}$$

We observe that $\lambda \approx n \cdot \Pi_F$ is the approximate parameter for a full sequence of transition, not per transition. Since the mean sequence length is $m_{SS}$ transitions, the expected count of transitions between visits to F is [4, 17], [25]:

$$m_{SS}E(\lambda) = m_{SS}\frac{\pi_S}{\pi_F} = m_{SS}\frac{m_{FF}}{m_{SS}} = m_{FF} \tag{17}$$

We exemplify this approach on the three state Markov chain (RMCM) given in Fig. 5. This RMCM corresponds to the Markov chain given in Fig. 4, where we added the fail state F associated to the states $C_i$, i = 1, 2, 3 associated to the classes in the Markov chain model for pulse width modulation scheme discussed in section 3.

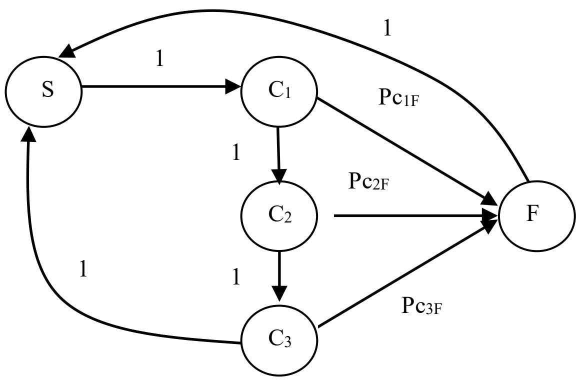


Fig.7. Five state RMCM associated to the Markov Chain in Fig.6

By colligating the RMCM in Fig. 5 with the Markov chain in Fig. 6, we notice the transition probabilities for the ordinary usage-states given in Fig. 6 ($p_{ij}$) and the small probabilities $Pc_{iF}$, i=1, 2, 3 in Fig. 7. Given that for highly reliable security communication systems we may have $0 \leq Pc_{iF} \leq 10^{-3}$, and correspondingly $0 < \Pi_{Fi} \leq 1{,}3 \cdot 10^{-4}$, which allow us to presume that for $Pc_{iF} \approx 10^{-4}$ the stationary distribution vector:

$$\Pi = [\pi_S, \pi_{C1}, \pi_{C2}, \pi_{C3}, \pi_F] = [0.14278,\ 0.42813,\ 0.14283,\ 0.14276,\ 1.42689 \cdot 10^{-5}] \tag{18}$$

The vector of mean recurrence time is:

$$[m_{SS}, ..., m_{FF}] = \left[\frac{1}{\pi_S}, ..., \frac{1}{\pi_F}\right] = [7.00400,\ 2.33412,\ 7.00120,\ 7.00470,\ 7.00821 \cdot 10^4] \tag{19}$$

The vector of the expected number of occurrences of states between transitions to non-rare state S is:

$$\frac{\pi}{\pi_S} = [1,\ 3.0009,\ 1.0005,\ 1.0005,\ 0.9989,\ 0{,}0005] \qquad (20)$$

The vector of the expected number of occurrences of states between transitions to rare state F is:

$$\frac{\pi}{\pi_F} = [1.0009,\ 3.0047,\ 1.0009,\ 1.0012,\ 0.0003] \cdot 10^4 \qquad (21)$$

The mean recurrence time of state S is $m_{SS} \approx 9$; therefore $n = 9 \cdot 10^4$ transitions correspond to approximatively $10^4$ average sequences from S back to S (e.g. we deal with reversible processes). The Poisson approximation of $L(n_F)$, see relation (11) for $n = 9 \cdot 10^4$ and $\lambda = 9\pi_F \cdot 10^4$ stands $p_{CF} = 10^{-4}$ and the rare state F has steady-state probability $\pi_F \approx 1.43 \cdot 10^{-5}$, and the upper bound $0{,}903 \cdot 10^{-5}$, where k = 0, 1, 2, 3. The MTTF is $m_{FF} \approx 9 \cdot 10^4$ for $p_{CF} = 10^{-4}$.

# 7 Conclusion

Our paper focuses on synthesizing results of randomized modulation with Markov chains, suitable for data transmitter, for example the ones used in telemedicine. Randomized modulation switching schemes governed by Markov chains applicable to d.c./a.c. or d.c./d.c. converters have been described.

Our representation for complex periodic Markov chains we believed to be novel. We also believe that this new model offers a new perspective for the spectral characteristics and other associated waveforms in a converter to the probabilistic structure that governs the dithering of an underlying deterministic nominal switching pattern.

Further research will continue to focus on minimization of one or multiple discrete harmonics. This approach corresponds to cases where the narrow-band characteristics corresponding to discrete harmonics are harmful, as for example in the telemedicine traffic security. We also discussed results in rare events for security communication system based on finite-state, discrete-parameter, recurrent Markov chain, here entitled reliability Markov chain model (RMCM). The chain provides a simple definition of failure as a rare event, respectively as a failure state F for which the steady-state $\Pi_F$ is orders of magnitude smaller than $\Pi_K$ for $k \neq F$, usually states of RMCM. Poisson law distribution bounds the transitions to a rare-fail state F in arbitrarily large size RMCM. Further research will focus on improvement of the analytic capabilities of RMCM in the study of extreme values of rare events [26, 27] when failure is infrequent and mean time to failure (MTTF) is long.

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